Linear algebra - Practice problems for midterm 2

1. Let $T : \mathcal{P}_2 \to \mathcal{P}_3$ be the linear transformation given by

$$T(p(x)) = \frac{dp(x)}{dx} - xp(x),$$

where $\mathcal{P}_2, \mathcal{P}_3$ are the spaces of polynomials of degrees at most 2 and 3 respectively.

(a) Find the matrix representative of T relative to the bases $\{1, x, x^2\}$ and $\{1, x, x^2, x^3\}$ for $\mathcal{P}_2$ and $\mathcal{P}_3$.

(b) Find the kernel of T.

(c) Find a basis for the range of T.

2. Determine whether the following subsets of $\mathcal{P}_3$ are subspaces.

(a) $U = \{p(x) \ : \ p(3) = 0\}$

(b) $V = \{p(x) \ : \ p(0) = 1\}$

(c) $W = \{p(x) \ : \ \text{the coefficient of } x^2 \text{ in } p(x) \text{ is } 0\}$.

3. Let $M_{m\times n}$ be the vector space of $m \times n$ matrices, with the usual operations of addition and scalar multiplication.

(a) Let A be an $m \times m$ matrix. Is the function

$$T : M_{m\times n} \to M_{m\times n}$$

given by $T(B) = AB$ a linear transformation?

(b) Let $V \subset M_{m\times n}$ be the subset consisting of those matrices, whose entries all add up to zero. Is V a subspace of $M_{m\times n}$?

4. Show that the subspaces $\text{sp}(x - x^2, 2x)$ and $\text{sp}(x^2, 3x + x^2)$ of $\mathcal{P}_2$ are equal.

5. Find a basis for the subspace $\text{sp}(1 + x^2, 2x - x^2, 4x + 2)$ of $\mathcal{P}_3$.

6. Working in the space $\mathcal{P}_3$, find the coordinate vector of x^2, relative to the basis $\{1, x - 1, (x - 1)^2, (x - 1)^3\}$.

7. Compute the determinant

$$\det \begin{bmatrix} 3 & -2 & 7 & 6 \\ -4 & 0 & 2 & 1 \\ 5 & 2 & 0 & -2 \\ 2 & 0 & -1 & 0 \end{bmatrix}$$

8. Suppose that A is an $n\times n$ matrix, such that all of the entries of A add up to zero. Is it true that $\det(A) = 0$?